

# One-Way ANOVA Repeated Measures

## Learning Objectives

- After the completion of this chapter, you will be able to
- Explain the concept of one-way ANOVA repeated measures
  - Describe the principle and assumptions of one-way ANOVA repeated measures
  - Design one-way ANOVA repeated measures
  - Explain how the research problem is formulated for one-way ANOVA repeated measures
  - Explain how the hypotheses are expressed in one-way ANOVA repeated measures
  - Demonstrate the steps used in SPSS to execute one-way ANOVA repeated measures
  - Explain how to analyse and interpret the SPSS outputs of one-way ANOVA repeated measures
  - Describe the important formulae and calculations of main ANOVA table
  - Report the final results of one-way ANOVA repeated measures

## 19.1 INTRODUCTION

In the previous chapters, we have discussed the applications of ANOVA which compares several population means of a dependent variable among more than two levels of independent variable (one-way ANOVA) and more than two independent variables (two-way ANOVA). Now, in this chapter, we extend our discussion to the application of one-way ANOVA in a way as repeated measures of observations. In this view, one-way ANOVA repeated measures is an experiment design in which subjects or objects are measured in a repeated manner aiming to ascertain whether a significant change occurs in the dependent variable.

One-way ANOVA repeated measures is a parametric test which compares three or more groups in the experiment consisting of the same subjects or objects. The number of repeated groups in the experiment is the same as number of treatments exposed (measured at nominal scale). The subjects or objects are same in each group. The variable of interest considered as dependent variable is similar among all the groups and measured at continuous scale (interval or ratio). Hence, repeated measures design is referred as an experiment in which all the subjects are exposed with more than one treatment and their effect is measured on each of the group rather than giving each level of treatment to different subjects (Kim 2015).

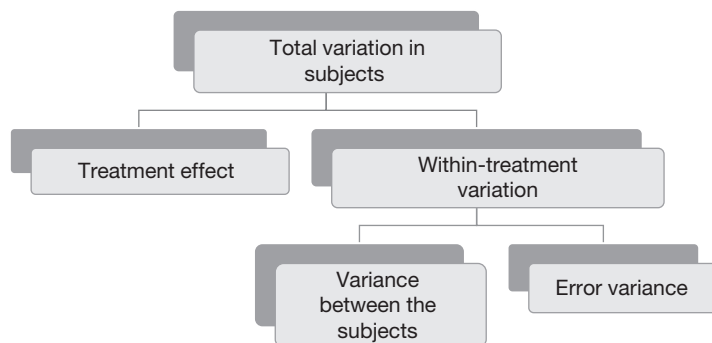
For example, we might be interested to investigate the most scoring pattern of exams among three different forms such as all MCQ, all subjective and mix-type (50% MCQ and 50% subjective) for undergraduate students. In this case, different examination patterns are considered for three distinct levels of treatment and all patterns are to be implicated to similar students one by one in a repeated manner at different times. Hence, we have three distinct groups of equal number of matched students who had undergone three different exam patterns. Thus, the score obtained by each group is measured at continuous scale. In this experiment, different patterns are to be assumed as independent variable with three levels (L-1: MCQ, L-2: Subjective and L-3: Mix-type), and the score of each group is considered as a dependent variable.

Application of one-way ANOVA repeated measures is employed in the situations where each individual subject or case is exposed to multiple levels of treatment of independent variable. The effect of these treatments is ascertained by exploring the significant difference among the group means, or in other words a same group of subjects is measured more than once for the same dependent variable.

## 19.2 PRINCIPLE OF ONE-WAY ANOVA REPEATED MEASURES

The source of variance in repeated measures ANOVA is subdivided into two broad categories. The variance in this ANOVA occurs due to different methods of experiment. This is same to the  $SS_{\text{between}}$  as applicable in one-way ANOVA. This source of variance is caused due to different conditions of an experiment design, hence also called  $SS_{\text{condition}}$ . Thus, in one-way ANOVA repeated measures,  $SS_{\text{between}} = SS_{\text{condition}}$ . The  $SS_{\text{condition}}$  represents the variance due to treatment effect in the experiment. The next source of variance is the amount of within-treatment variation that is not explained by the experiment manipulation ( $SS_{\text{within-subject}}$ ). This is same as we discussed in error variance ( $SS_{\text{error}}$ ) of one-way ANOVA and is caused due to within-subjects. Thus, in one-way ANOVA repeated measures,  $SS_{\text{error}} = SS_{\text{within-subjects}}$ . It is important to note that as in repeated measures, the subjects are repeated in the experiment. Hence, the  $SS_{\text{within-subjects}}$  is again better explained by partitioning into difference in variance between the subjects ( $SS_{\text{between}}$ ) considering each subject group as block and unexplained variance also referred as error variance due to within-subject. The partition of total variability is schematically presented as shown in Figure 19.1.

Figure 19.1 Partitioning the Variance: One-Way ANOVA Repeated Measures



The repeated measures design is also called within-subjects analysis as the scores of same cases among all the treatment groups are dependent (the result of a particular student in all three types of exam pattern), whereas the scores of different cases among the groups (score of two students in all three types of exam pattern) are independent of each other. Mean sum of square (MS) for conditions ( $MS_{\text{conditions}}$ ) and error ( $MS_{\text{error}}$ ) are computed by dividing appropriate df, and finally  $F$ -statistics represents the ratio of variance explained by the experiment manipulation to the error variance. It is important to note that due to partition of error term in one-way ANOVA repeated measures, the denominator of  $F$ -ratio is reduced substantially and results into more power to detect the significant mean difference among the group subjects as compared to one-way ANOVA standard test.

**Table 19.1 Source of Variance: One-Way ANOVA Repeated Measures**

Source of Variance	Cause of Variability	Degree of Freedom	Sum of Squares (SS)	Mean Sum of Squares (MS)	F-statistics (or F-ratio)
Conditions variance	Different conditions of treatment	No. of conditions – 1 ( $k-1$ )	$SS_{\text{conditions}}$	$MS_{\text{conditions}}$	$MS_{\text{conditions}} / MS_{\text{error}}$
Subject variance	Due to individual subject/case	Total subjects – 1 ( $n-1$ )	$SS_{\text{subjects}}$	$MS_{\text{subject}}$	
Error variance	Unexplained	$(k-1) \times (n-1)$	$SS_{\text{error}}$	$MS_{\text{error}}$	

### 19.3 ASSUMPTIONS FOR ONE-WAY ANOVA REPEATED MEASURES

Exploring the findings with one-way ANOVA repeated measure is based on the following assumptions and design considerations. These assumptions are required to verify so as to ensure the validity of the results for the target population:

- **Normal distributed dependent variable:** The variable assumed to be dependent in the experiment is normal distributed among all the matched pairs. Certain statistical tests such as KS one-sample test, Shapiro–Wilk, measures of skewness and kurtosis are used to examine the normality of data. Whereas, histogram, box and whisker, Q–Q and P–P plots are used as graphical displays to confirm the normality of data.
- **Homogeneity of variance:** Another important assumption is the equal variance distribution in the target population. This assumption is also called sphericity assumption or homogeneity-of-variance-of-difference. In order to examine this assumption, SPSS computes Mauchly’s test of sphericity to verify the equal variance of difference among the repeated measures factors in experiment.
- **Randomly selected sample:** A random selection of sample must be ensured from the target population. The observations are independent of each other, that is, value of one observation is not related to other observation, and thus the error terms are uncorrelated in the population. The scores of dependent variables for same individual in different conditions (experiment levels) are not related to each other.
- **Data scale:** A scale of dependent variable(s) is an important criterion for the validity of the experiment. The dependent variable should be measured at continuous scale

interval or ratio. Whereas, the independent variable should be categorical in nature with at least three distinct levels considering as related groups or matched pairs.

The assumptions are summarized in Table 19.2.

**Table 19.2 Assumptions: One-Way ANOVA Repeated Measures**

<i>Assumptions</i>	<i>Examine Approach</i>	<i>Measures/Tests</i>
Normal distribution of all matched pairs	Graphical method	Histogram and box and whisker plot
	Numerical method	Skewness and kurtosis
		KS One-sample and Shapiro–Wilk
Homogeneity of variance	Numerical method	Levene’s test
Independent observations at different conditions	Methodological	Random selection of sample
Matched pairs	Methodological	Nominal based independent variable with related groups
Absence of outliers	Graphical method	Box and whisker plot

## 19.4 DESIGNING EXPERIMENT

The experiment design in one-way ANOVA repeated measures investigates about changing the mean scores over different time periods or due to three or more different conditions (Level-1 to Level-k). In both the cases, the same subjects ( $S_1$ – $S_n$ ) are being measured repeatedly for common variable of interest. The levels of independent variable in both the cases are called within-subject factor. When dependent variable is measured over a specific time period, the levels represent time, and when measurements are done for different conditions, the levels indicate particular conditions. Hence, blocking and condition means are calculated for analysing the variance. The designing of the experiment must ensure to repeat the same subjects with exposing all the conditions as shown in Table 19.3.

**Table 19.3 Experiment Design: One-Way ANOVA Repeated Measures**

	<i>Time/Conditions (j = 1, 2, ..., k)</i>				<i>Blocking Mean</i>
	<i>Level-1</i>	<i>Level-2</i>	<i>Level-3</i>	<i>Level-k</i>	
Matched subjects	$S_{11}$	$S_{21}$	$S_{31}$	$S_{k1}$	$\bar{X}_{1i}$
	$S_{12}$	$S_{22}$	$S_{32}$	$S_{k2}$	$\bar{X}_{2i}$
	$S_{13}$	$S_{23}$	$S_{33}$	$S_{k3}$	$\bar{X}_{3i}$
	$S_{1n}$	$S_{2n}$	$S_{3n}$	$S$	$\bar{X}_{ni}$
Conditions means	$\bar{X}_{j1}$	$\bar{X}_{j2}$	$\bar{X}_{j3}$	$\bar{X}_{jk}$	$\mu$ (overall mean)

## 19.5 FRAMING HYPOTHESIS

Hypothesis designing in one-way ANOVA repeated measures aims to determine whether the treatment means of matched pairs are same. As discussed earlier, the ‘null hypothesis’ specifies the condition of equality and assumes that each group has equal mean. The alternative hypothesis indicates significant difference in means among various treatment groups due to the difference occurring in at least two of the conditions. Symbolically, both the hypotheses can be expressed as follows:

Null hypothesis  $H_0$ :  $\mu_1 = \mu_2 = \mu_3 \dots \mu_k$  ( $k$  = number of levels)  
(The population mean is same among all matched pairs.)

We can also express the null hypothesis associated to each level of independent variable revealing different conditions.

$H_0$ :  $\mu_{\text{condition 1}} = \mu_{\text{condition 2}} = \mu_{\text{condition 3}} \dots \mu_{\text{condition } k}$

Alternative hypothesis  $H_a$ :  $\mu_1 \neq \mu_2 \neq \mu_3 \neq \dots \mu_k$   
(The population mean is not same in all the matched pairs.)

## 19.6 RESEARCH PROBLEM AND TEST TECHNIQUE

This chapter is based on a hypothetical research problem aiming to understand the concept and process of one-way ANOVA repeated measures. In view of this, data file `cake_recipe.sav` presents the rating of taste preference (1 = least preferred to 10 = most preferred) of three different cakes among 40 randomly selected customers. The cakes are prepared from three different recipes including creaming method, one-bowl method and cream-and-whip-method. In this experiment, different methods are to be assumed as independent variable depicting three levels (L-1: creaming method, L-2: one-bowl method and L-3: cream-and-whip-method) and served to the same group of customers (40 in number) one by one in a repeated manner. Hence, an investigation is proposed to measure the difference in preference scores (dependent variable) among three matched groups. At this case, as we are dealing with one independent variable (three distinct conditions due to different cake recipes) and one dependent variable (preference scores) along with measuring the dependent variable for three times from matched pairs, one-way ANOVA repeated experiment design is employed in order to investigate whether taste preference is same for different cakes as indicated by different recipes.

## 19.7 EXECUTING ONE-WAY ANOVA REPEATED MEASURES WITH SPSS

By using data file `cake_recipe.sav`, we will measure the difference in taste preference of three different kinds of cake recipes among certain groups of customers in a repeated manner. To design this experiment, the taste preferences of 40 randomly selected customers are measured thrice in a repeated manner. General Linear Model (GLM) Univariate Analysis is used for executing repeated measures in SPSS. The various commands associated in this execution

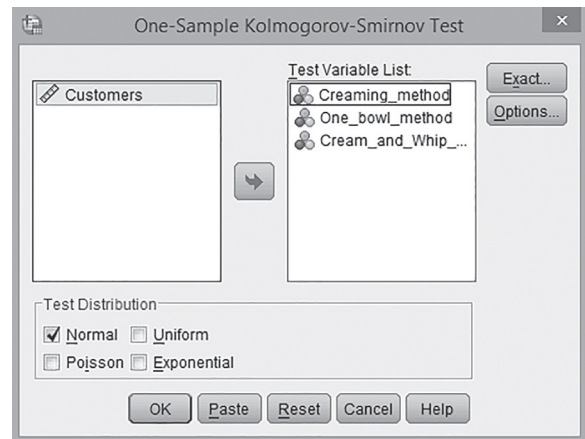
process will be used to compute descriptive statistics for different conditions—Mauchly’s test of sphericity for equality of variance, ANOVA table for tests of within-subjects factors, tests of within-subjects contrasts, plotting line graphs for interpreting mean difference and post hoc analysis for breaking down the effects in separate SPSS outputs. The following steps are used to compute one-way ANOVA repeated measures.

### 19.7.1 Assessing Normality

As we explained earlier, one-sample Kolmogorov–Smirnov test is employed to examine the normality of data for three different conditions. This is an assumption for applying ANOVA application. The steps used for the same are shown in Exhibit 19.1.

**Exhibit 19.1.** Use cake\_recipe.sav » Menu bar » Analyse » Non-parametric test » Legacy Dialogs » One-Sample KS test » Transfer creaming method, one-bowl method and cream-and-whip-method to test variable(s) list » Click on normal distribution under test distribution » Press OK

**Figure 19.2 Main Dialog Box: One-Sample KS Test for Repeated Designs**



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The dialog box appears on the screen as shown in Figure 19.2.

The output viewer from SPSS shows the results of one-sample KS test as shown in Table 19.4.

**Table 19.4 Assessing Normality with One-Sample KS**

Recipe Methods	N	Normal Parameters		KS Z	Sig. (2-tailed)
		Mean	Std Deviation		
Creaming	40	4.1	2.4	0.134	0.070
One-bowl	40	6.0	2.3	0.121	0.146
Cream-and-whip	40	5.6	2.3	0.116	0.186

Table 19.4 presents the mean score of three methods (creaming = 4.1, one-bowl = 6.0 and cream-and-whip = 5.6) along with almost same standard deviation (2.3) for 40 subjects. The *p*-values (Sig. [2-tailed]) as shown in the last column of these observations are more than 0.05

( $p > 0.05$ ,  $p = 0.070$ ,  $0.146$  and  $0.186$ ) for all the three measurements at 5% LoS; hence, we fail to reject the null hypothesis indicating the condition of normality and conclude that the observations of taste preference of all the three methods is normally distributed in the target population. Thus, the data can be used for employing repeated measures ANOVA.

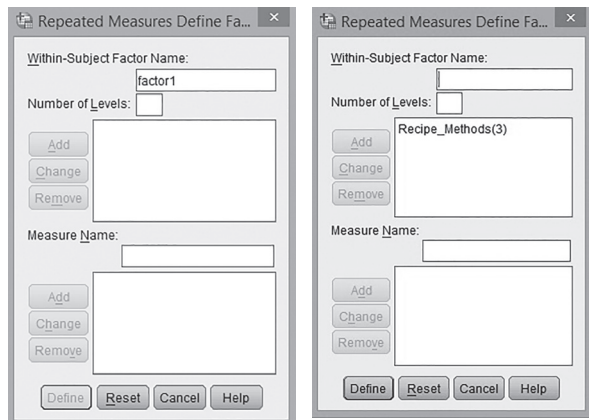
## 19.7.2 SPSS Path for Specifying Repeated Measures Factor

Specifying the names and assigning the levels of repeated measure factor as mentioned in data set is the first task. The steps as mentioned in Exhibit 19.2 are used for the same purpose.

**Exhibit 19.2.** Use `cake_recipe.sav` » Menu bar » Analyse » General linear model » Repeated measures » Type `Recipe_Methods` as factor name and type 3 as number of level » Click add button » Define

The dialog box as shown in Figure 19.3 appears on the screen representing GLM Repeated Measures by using the steps as mentioned in Exhibit 19.2. In our case, we mentioned *Recipe\_Methods* as within-subject factor name.

**Figure 19.3** General Linear Model Repeated Measures: Main Dialog Box



## 19.7.3 SPSS Path for Mentioning Levels of Independent Factor

The next path is used to mention the levels of within-subject factor by using the following steps mentioned in Exhibit 19.3 in continuity of Exhibit 19.2.

**Exhibit 19.3.** Click Define » Select `Creaming_method`, `One_bowl_method` and `Cream_and_whip_method` (either select one-by-one or all by pressing the shift key) » Transfer to Within-Subject Variables box

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By using the steps as mentioned in Exhibit 19.2, three distinct levels of within-subject factor, namely creaming, one-bowl and cream-and-whip-methods are transferred to the variable box for further analysis. The box appears on the screen as shown in Figure 19.4.

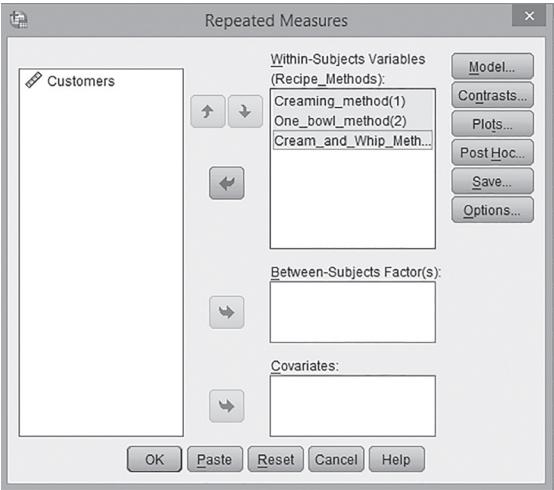
### 19.7.4 SPSS Path for Computing Profile Plot

The next path is about creating profile plot in order to display the change in the dependent variable and interpret the mean difference among various treatments. A single line graph is plotted by using the following steps mentioned in Exhibit 19.4 in continuity of Exhibit 19.2.

**Exhibit 19.4.** Click Plots » Select Recipe\_methods » Transfer in horizontal axis » Click on Add button

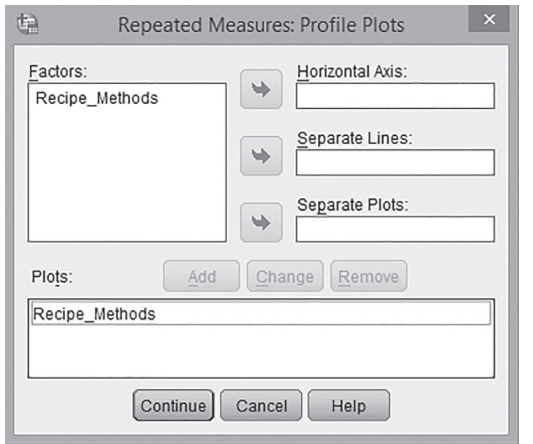
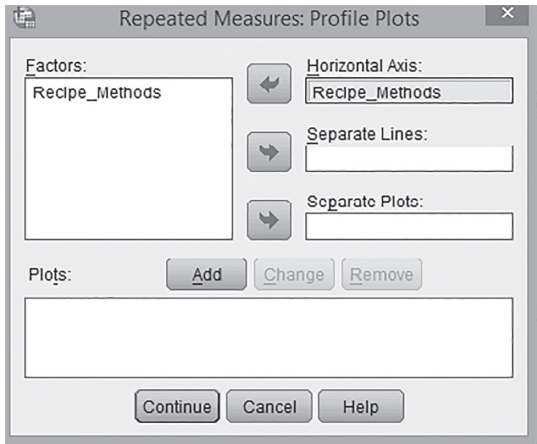
The mean difference of taste preference for three different recipe methods (creaming method, one-bowl method and cream-and-whip-method) is displayed in single line chart. Estimated marginal means of the dependent variable and categories of the independent factor are displayed on Y- and X-axis, respectively. The screen box as shown in Figure 19.5 appears by using the steps mentioned in Exhibit 19.4.

**Figures 19.4 Repeated Measures Main Dialog Box: Define**



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**Figure 19.5 Repeated Measures Main Dialog Box: Profile Plots**



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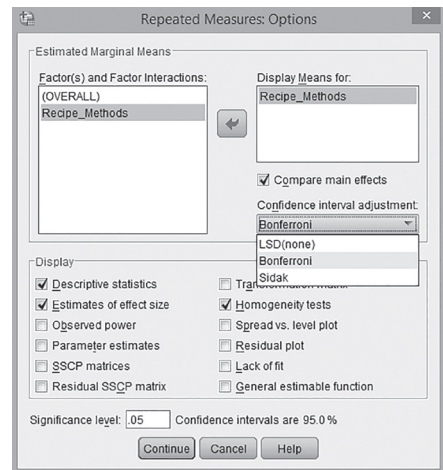
### 19.7.5 SPSS Path for Computing Descriptive Statistics, Homogeneity Test and Effect Size

We need to follow the below-mentioned steps in continuity of Exhibit 19.2 in order to compute the descriptive statistics for different treatments, Mauchly's test of Sphericity for verifying the assumption of equal variance and estimating the effect size.

**Exhibit 19.5.** Click Options of repeated measures dialog » Transfer *Recipe\_methods* in box displaying means for » Select compare main effects » Select bonferroni test from drop menu list of confidence interval adjustment » Select descriptive statistics, homogeneity test and estimates of effect size in display column » Continue » Press OK

The dialog box appears on the screen as shown in Figure 19.6 by using the steps mentioned in Exhibit 19.5.

**Figure 19.6** Repeated Measures Main Dialog Box: Options



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## 19.8 INTERPRETING OUTPUTS OF ONE-WAY ANOVA REPEATED MEASURES

The output viewer from SPSS shows the results of one-way ANOVA repeated measures as shown in Tables 19.5a through 19.5f.

### 19.8.1 Identification of Labels

The first output of repeated measures ANOVA indicates the labels of the independent variable as mentioned in data set. Table 19.5a presents three recipe methods in the sequence of creaming method, one-bowl method and cream-and-whip method. Here, the dependent variable is assumed as taste preference of subjects for three distinct recipe methods labelled from 1 to 3.

**Table 19.5a** Within-Subjects Factors Measures: One-Way ANOVA Repeated Measures

<i>Recipe Methods</i>	<i>Dependent Variable</i>
1	Creaming method
2	One-bowl method
3	Cream-and-whip method

## 19.8.2 Descriptive Statistics between Groups

Table 19.5b reveals the descriptive statistics and shows the characteristics of location and spread by indicating mean and standard deviation. It can be observed that taste for one-bowl method is highly preferred (6.0) by the respondents followed by cream-and-whip method (5.6) and creaming method (4.1). The standard deviation is almost same (2.3) for all the three methods.

**Table 19.5b Descriptive Statistics: One-Way ANOVA Repeated Measures**

	<i>Mean</i>	<i>Std Deviation</i>	<i>N</i>
Creaming_method	4.1	2.4	40
One_bowl_method	6.0	2.3	40
Cream_and_whip_method	5.6	2.3	40

## 19.8.3 Mauchly's Test of Sphericity

Table 19.5c indicates the results of Mauchly's test of sphericity. This test is used to examine the assumption of equal variance as we have discussed the application of Levene's test in ANOVA. The sphericity is referred to a condition of equal variances of the difference between all the possible pairs of matched groups (i.e., different levels of independent variable) in the experiment (Field 2009). This test is used to check the assumption whether each of the recipe method is approximately equally correlated with the other scores.

Non-significant  $p$ -value associated with Mauchly's test of sphericity fulfils the assumption of equal variance. As the  $p$ -value (0.381) of this test statistic is non-significant (more than 0.05 at 5% LoS), we fail to reject the null hypothesis of equal sphericity and conclude with strong evidence that sphericity is assumed to be the same across the matched pairs in the experiment. The Epsilon measures at right side of the table indicating three different ways for adjustment in  $df$  to calculate  $F$ -statistics in the condition of violation in the assumption of equal sphericity. In non-significant  $p$ -value of Mauchly's test of sphericity, use  $p$ -value of Greenhouse-Geisser correction row for checking the assumption.

**Table 19.5c Mauchly's Test of Sphericity**

<i>Mauchly's Test of Sphericity<sup>a</sup></i>							
<i>Within-Subjects Effect</i>	<i>Mauchly's W</i>	<i>Approx. Chi-Square</i>	<i>df</i>	<i>Sig.</i>	<i>Epsilon<sup>b</sup></i>		
					<i>Greenhouse-Geisser</i>	<i>Huynh-Feldt</i>	<i>Lower-bound</i>
Recipe methods	0.951	1.928	2	0.381	0.953	1.000	0.500

Notes: <sup>a</sup>Design-Intercept (Within-Subject Design-Recipe\_Method).

<sup>b</sup>May be used to adjust the degrees of freedom for averaged test of significance.

### 19.8.4 Tests of Between-Subjects Effects and Calculations for Sum of Squares, df, Mean Sum of Squares, F-Ratios and Partial Eta Squared

Table 19.5d presents the main table of repeated measures ANOVA aiming to drawing evidence for statistical significance of mean difference among the target population for different conditions as poised in the experiment. The source of variance in repeated measures ANOVA is subdivided into two broad categories including the variance due to different recipe methods ( $SS_{\text{between}}$ ) of the experiment and the amount of within-treatment variation that is not explained by the experiment manipulation ( $SS_{\text{error}}$ ). As sphericity is assumed in data, we will consider the  $F$ -statistic (6.6) and associated  $df$  (2, 78) indicating the row of sphericity assumed condition. The mean sum of squares are computed by dividing the sum of squares (77.2) with respective  $df$  (2), and finally  $F$ -statistic (6.6) is computed by dividing mean sum of square (38.6) with error mean square (5.7). The rest of the columns represent probabilistic values and partial eta squared.

The  $p$ -values as associated with  $F$ -statistic in the condition of sphericity assumed [ $F$  (2,78) = 6.6,  $p < 0.05$ , 0.002] is less than 5% LoS. Hence, we reject the null hypothesis of equal taste preference for all three methods and would have evidence that significant difference exists in the taste preference between at least any two methods.

**Table 19.5d Tests of Within-Subjects Effects: One-Way ANOVA Repeated Measures**

Source		Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Recipe methods	Sphericity assumed	77.2	2	38.6	6.6	0.002	0.146
	Greenhouse–Geisser	77.2	1.9	40.5	6.6	0.003	0.146
	Huynh–Feldt	77.2	2.0	38.6	6.6	0.002	0.146
	Lower-bound	77.2	1.0	77.2	6.6	0.014	0.146
Error (recipe methods)	Sphericity assumed	451.4	78	5.7			
	Greenhouse–Geisser	451.4	74.3	6.0			
	Huynh–Feldt	451.4	78.0	5.7			
	Lower-bound	451.4	39.0	11.5			

The below presented calculations are required to comprehend and interpret the main ANOVA table.

#### I. Computing sum of squares

- i. **Sum of square between conditions ( $SS_{\text{condition}}$ ):** As we have three conditions (recipe methods) and 40 cases under each condition, three mean scores belonging to each condition (creaming method = 4.1, one-bowl method = 6.0 and cream-and-whip method = 5.6) and grand mean (5.29) are used to compute the  $SS_{\text{condition}}$ . It is computed by comparing the mean of each matched pair with the grand mean. The difference is squared and weighted by the number of subjects in the respective

group, and finally all the individual results are summed. The following formula is used to calculate  $SS_{\text{condition}}$ :

$$\sum_{i=1}^K n_i (\bar{X}_i - \bar{X})^2,$$

where  $\bar{X}_i$  represents sample mean of each condition (*ith*),  $\bar{X}$  the overall mean of the experiment and  $n_i$  the number of observations being measured in each condition.

$$\text{Thus, } SS_{\text{condition}} = 40 (4.1-5.29)^2 + 40 (6.0-5.29)^2 + 40 (5.6-5.29)^2 = 77.2.$$

- ii. **Sum of square within sample ( $SS_{\text{within}}$ ):**  $SS_{\text{within}}$  can be obtained as sum of squared deviation from each score to its corresponding treatment or condition group. It represents within-subjects variation in the sample and is computed as follows:

$$\sum_{i=1}^n \sum_{j=1}^k (\dot{X}_{ij} - \bar{X}_j)^2,$$

where  $n$  is the number of observations recorded,  $X_{ij}$  is the *ith* observation of *jth* condition and  $\bar{X}_j$  represents sample mean for condition.

$$\begin{aligned} \text{Thus, } SS_{\text{within}} &= [(5-4.1)^2 + (6-4.1)^2 + \dots + (3-4.1)^2 + (9-4.1)^2] + [(6-6.0)^2 + (4-6.0)^2 \\ &+ \dots + (7-6.0)^2 + (6-6.0)^2] + [(5-5.6)^2 + (6-5.6)^2 + \dots + (3-5.6)^2 + (4-5.6)^2] \\ &= 669.74 \end{aligned}$$

- iii. **Sum of square between subject ( $SS_{\text{subject}}$ ):**  $SS_{\text{subject}}$  can be obtained as sum of squared deviation of subject mean from grand mean and then multiplied with the number of conditions in the experiment. As mentioned earlier also, each subject mean is treated as blocking variable representing as a level of independent variable. Hence, following formula is used:

$$j * \sum (\bar{X}_{ij} - \bar{X})^2$$

where  $\bar{X}_i$  represents subject mean for all condition,  $\bar{X}$  the overall mean of the experiment and  $j$  is the number conditions employed in the experiment.

$$\text{Thus, } SS_{\text{subject}} = 3 \times [(5.33-5.29)^2 + (5.33-5.29)^2 + \dots + (4.33-5.29)^2 + (6.33-5.29)^2] = 218.1$$

- iv. **Computing error sum of square ( $SS_{\text{error}}$ ):**  $SS_{\text{error}}$  represents the amount of within-treatment variation not explained by the experiment manipulation. It is computed as follows:

$$SS_{\text{error}} = SS_{\text{within}} - SS_{\text{subject}}$$

$$\text{Thus, } SS_{\text{Error}} = 669.74 - 218.1 = 451.64.$$

## II. Computing df

df for three sources of variance—conditions, subjects and error in the experiment can be computed as shown in Table 19.6.

**Table 19.6 Degrees of Freedom: One-Way ANOVA Repeated Measures**

Variation Source	Formulae for Degree of Freedom (df)	df
Conditions	Number of conditions-1, (k-1)	3-1 = 2
Subjects	Number of subject-1, (N-1)	40-1 = 39
Error variance	(k-1) × (N-1)	2 × 39 = 78

**III. Computing mean sum of squares and F-statistics**

By dividing the sum of squares by corresponding df, we can get mean sum of square for each main and interaction effects. Finally, three *F*-ratios can be computed by dividing the mean sum of square by residual mean squares as shown in Table 19.7.

**Table 19.7 F-Ratios: One-Way ANOVA Repeated Measures**

Mean Sum of Square (MS)	Value	F-Ratio	Value
MS = SS/2	77.2/2 = 38.6	$F = MS/MS_{\text{Error}}$	38.6/5.78 = 6.6
MS <sub>Error</sub> = SS <sub>Error</sub> /78	451.4/78 = 5.78		

**IV. Computing effect size**

Partial eta squared ( $\eta^2_p$ ) in repeated measures ANOVA indicates the proportion of variance in dependent variable accounted for by the independent variable. Partial eta squared can be calculated by dividing variance explained with conditions (SS<sub>condition</sub>) by sum of SS<sub>condition</sub> and error variance (SS<sub>error</sub>). It ranges from 0 to 1. Table 19.8 indicates the calculations of  $\eta^2_p$  as computed by SPSS.

**Table 19.8 Effect Size: One-Way ANOVA Repeated Measures**

Partial Eta Squared	Formulae	Value
$\eta^2_p$	SS <sub>Condition</sub> /(SS <sub>condition</sub> + SS <sub>error</sub> )	77.2/17.2 + 451.4 = 0.16

The interpretation of  $\eta^2_p$  is same as coefficient of determination ( $R^2$ ). The value of partial eta squared indicates that approximately (0.16 × 100 = 16%) 16% of the variance in taste preference (dependent variable) can be attributed due to different recipe methods (independent variable) in the experiment.

**19.8.5 Pairwise Comparison**

Table 19.9 presents the results of pairwise comparison of mean difference in taste preferences among all the possible pairs. The CI for difference is adjusted with *Bonferroni* measures as we have selected this option during the steps as shown in Exhibit 19.4. As explained earlier, on the basis of *p*-value (at 5% LoS), we have statistical evidence in support for not to reject the

null hypothesis of equal taste preference for only one pair (2 and 3; one bowl and cream-and-whip-method) among all possible pairs due to higher  $p$ -values. These pairs have minimum mean difference (0.350) as compared to rest of the pairs. Noticing a particular label (1 = creaming method, 2 = one-bowl method and 3 = cream-and-whip-method) of recipe method is important to consider for valid comparison. If ANOVA result seems insignificant, then applying multiple comparison is meaningless due to no-difference exists among all the possible pairs.

**Table 19.9 Pairwise Comparison: One-Way ANOVA Repeated Measures**

(I) Methods	(J) Methods	Difference (I-J)	Std Error	Sig. <sup>b</sup>	95% CI for Difference <sup>b</sup>	
					Lower Bound	Upper Bound
1	2	-1.850 <sup>a</sup>	0.544	0.005	-3.210	-0.490
	3	-1.500 <sup>a</sup>	0.480	0.010	-2.702	-0.298
2	1	1.850 <sup>a</sup>	0.544	0.005	0.490	3.210
	3	0.350	0.585	1.00	-1.112	1.812
3	1	1.500 <sup>a</sup>	0.480	0.010	0.298	2.702
	2	-0.350	0.585	1.00	-1.812	1.112

Notes: Based on estimated marginal means:

<sup>a</sup>The mean difference is significant at the 0.05 level.

<sup>b</sup>Adjustment for multiple comparisons: Bonferroni.

## 19.9 SAMPLE DATA WITH REPEATED OBSERVATIONS

Sample data as presented in Table 19.10 is important to comprehend the manual calculations for the results of repeated measure ANOVA within-subjects effects. The table indicates the mean scores ( $\bar{X}_{\text{Creaming}} = 4.1$ ,  $\bar{X}_{\text{One-bowl}} = 6.02$  and  $\bar{X}_{\text{Cream-and-Whip}} = 5.6$ ) of taste preference for three different recipe methods. The means score is obtained from 40 subjects in repeated manner. The table also presents the mean score of subject-wise and is considered as a blocking mean in the experiment as depicted in the last column. The blocking mean acts the same as the level of the independent variable.

**Table 19.10 Sample-Data: One-Way ANOVA Repeated Measures**

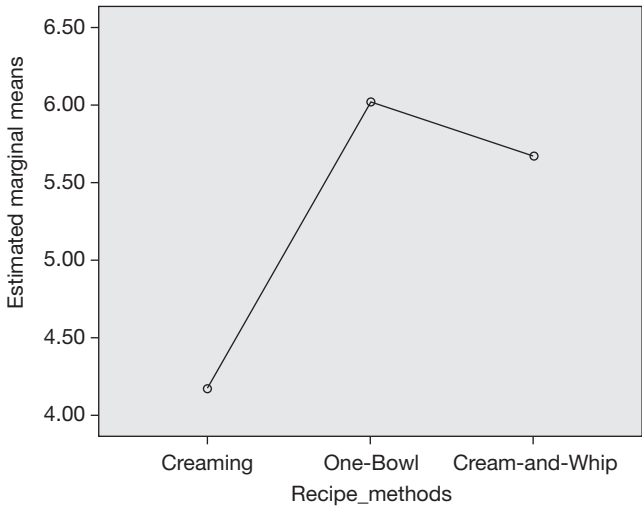
Subjects	Conditions-Recipe Methods			Subject Means (Blocking Mean)
	Creaming	One-bowl	Cream-and-Whip	
1	5	6	5	5.33
2	6	4	6	5.33
3	4	8	3	5.00
4	5	9	3	5.67
5	1	5	2	2.67

Subjects	Conditions-Recipe Methods			Subject Means (Blocking Mean)
	Creaming	One-bowl	Cream-and-Whip	
35	3	9	7	6.33
36	2	9	8	6.33
37	5	2	5	4.00
38	8	9	3	6.67
39	3	7	3	4.33
40	9	6	4	6.33
	$\bar{X}_1 = 4.1$	$\bar{X}_2 = 6.02$	$\bar{X}_3 = 5.6$	
Grand mean ( $\bar{X}$ ) = 3.01				

### 19.10 DISPLAYING ESTIMATED MARGINAL MEANS

The final part of the output indicates the visual display of estimated marginal means of taste preference in a single line chart. As we have selected the profile plot in the steps as mentioned in Exhibit 19.3, the line chart appears in the output window. The estimated marginal means of *taste preference* is displayed at vertical (Y) axis, whereas three recipe methods are at horizontal (X) axis. This kind of chart appeared as the variable *Recipe\_Methods* transferred into horizontal axis of profile plot in the repeated measures main dialog box (Exhibit 19.4). We can easily interpret a significant difference in the taste preference among all the methods as shown in Figure 19.7.

**Figure 19.7 Displaying Interaction between Factors**



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## 19.11 REPORTING THE RESULTS OF ONE-WAY ANOVA REPEATED MEASURES

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One  $F$ -statistic with respect to whether sphericity assumed or not, associated df (whether adjusted or not), corresponding  $p$ -values, pairwise comparison among all possible combinations, magnitude of partial eta squared, and graphical display of estimated marginal means are the important parts of the final results.

- As the  $p$ -values associated with  $F$ -statistic in sphericity assumed condition [ $F(2,78) = 6.6$ ,  $p < 0.05$ , 0.002] is less than 5% LoS, we reject the null hypothesis of equal taste preference for all the three methods and would have evidence that a significant difference exists at least between any two methods.
- The results of pairwise comparison of main effect form the measures of Bonferroni CI adjustment indicating that null hypothesis of equal taste preference fails to reject only one pair (2 and 3; one-bowl and cream-and-whip-methods) among due to higher  $p$ -value ( $p > 0.05$ , 1.00) at 5% LoS.
- The measure of partial eta squared indicates that 16% proportion of variance in the taste preference is explained by the factors incorporated in the experiment.

### SUMMARY

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- **Concept of one-way ANOVA repeated measures:** The one-way ANOVA repeated measures is a parametric test procedure used to compare three or more groups in the experiment consisting of the same subjects or cases. The number of repeated groups in the experiment is same as number of treatments exposed. The subjects or objects are also same in each group.
- **Principle of one-way ANOVA repeated measures:** The source of variance in one-way ANOVA repeated is caused due to different conditions of an experiment design. This source of variance is also known as  $SS_{\text{condition}}$ . Thus, in one-way ANOVA, repeated measures  $SS_{\text{between}}$  is equal to  $SS_{\text{condition}}$ . This  $SS_{\text{condition}}$  represents the variance due to treatment effect in the experiment. Second, the source of variance is the amount of within-treatment variation ( $SS_{\text{within-subject}}$ ). This is the same as the error variance ( $SS_{\text{error}}$ ) of one-way ANOVA and caused due to within subjects.
- **Assumptions of one-way ANOVA repeated measures:** The normal distributed dependent variable, equal variance distribution in target population, random selection of samples and metric and non-metric (nominal scale) scale of dependent and independent variables (different conditions) are the main assumptions of one-way ANOVA repeated measures.
- **Experiment design:** The experiment design in one-way ANOVA repeated measures indicates about changing the mean scores over different time periods or due to three or more different conditions. In both cases, the same subjects or respondents are measured for common variable of interest in a repeated manner.
- **Framing hypotheses:** Hypothesis designing in one-way ANOVA repeated measures aims to determine whether the treatment means of matched pairs are same.



- **Research problem in one-way ANOVA repeated measures:** The research problem aims to measure the difference in the dependent variable among distinct matched groups. The independent variable indicates at least three distinct conditions due to treatment and one dependent variable is measured from matched pairs.
- **Steps used in SPSS to execute one-way ANOVA repeated measures:** The following steps are used in SPSS to execute one-way ANOVA repeated measures:

Examine normality of dependent variable in matched sample with KS one-sample test  
 » Specify repeated measures factor » Mentioning levels of independent factor »  
 Computing profile plot » Computing descriptive statistics, homogeneity test and effect size  
 » Tests of between-subjects effects » Post hoc analysis » Reporting final results

- **Report the final results of one-way ANOVA repeated measures:** The  $F$ -ratio with respect to whether sphericity assumed condition, associated  $df$  (whether adjusted or not), corresponding  $p$ -values, pairwise comparison, magnitude of partial eta squared, and graphical display of estimated marginal means are the important parts.

## KEY TERMS

Blocking mean	Matched case
Conditions variance	Mauchly's test of sphericity
Effect size	Partial eta squared
Epsilon measures	Profile plots
Error variance	Repeated measures
Eta squared	Treatment effect
General linear model univariate analysis	Unexplained variance
Greenhouse–Geisser correction	
Marginal means	

## Solved Example

- Q1.** Ten participants are selected in a final round of 'Master-Chef' competition. The participants are asked to prepare three recipes A, B and C. Each participant prepared three recipes separately and their respective time of preparation (in minutes) is recorded on individual basis for all the three recipes. The following data presents this observation:

Participants	Time in Preparation (minutes)		
	Recipe A	Recipe B	Recipe C
1	35	67	85
2	45	87	90
3	50	56	75

4	55	80	70
5	43	55	69
6	54	60	58
7	34	56	60
8	47	76	70
9	39	59	65
10	42	87	89

**Answer the following questions:**

- Prepare data set in SPSS as per the experiment of one-way ANOVA repeated by using proper scale and data.
- Examine the assumption of sphericity in the experiment.
- Comment on the statistical difference of preparation time among participants for three methods.

**Solution**

- Three methods of recipes (A, B and C) are considered as treatment and time preparation from each individual for all recipe methods is considered as test variable. Thus, three variables *RecipeA*, *RecipeB* and *RecipeC* are created in variable view and time taken for each method is filled in data view as per the given data. The data view can be seen as shown in Figure 19.8.

**Figure 19.8 Data View: One-Way ANOVA Repeated Measures**

	RecipeA	RecipeB	RecipeC
1	35.00	67.00	85.00
2	45.00	87.00	90.00
3	50.00	56.00	75.00
4	55.00	80.00	70.00
5	43.00	55.00	69.00
6	54.00	60.00	58.00
7	34.00	56.00	60.00
8	47.00	76.00	70.00
9	39.00	59.00	65.00
10	42.00	87.00	89.00

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- The non-significant  $p$ -value ( $p > 0.05$ , 0.453) as revealed in Mauchly's test at 5% LoS confirms the assumption of equal variance as shown in Table 19.11a. Hence, the condition of sphericity is assumed for the given experiment.

Table 19.11a Mauchly's Test: Solution

Within-Subjects Effect	Mauchly's <i>W</i>	Approx. Chi-Square	<i>df</i>	Sig.	Epsilon <sup>b</sup>		
					Greenhouse–Geisser	Huynh–Feldt	Lower-bound
Recipe	0.812	1.66	2	0.435	0.842	1.00	0.50

Note: <sup>b</sup>May be used to adjust the degrees of freedom for averaged test of significance.

- c. The *p*-values as mentioned in *F*-statistic belonging to the condition of sphericity assumption [ $F(2,18) = 28.7, p < 0.05, 0.000$ ] are less than 5% LoS as shown in Table 19.11b. Hence, we reject the null hypothesis of equal time preparation for all three recipe methods and would have evidence that significant difference exists in the time preparation between any two recipe methods.

Table 19.11b Tests of Within-Subjects Effects: Solution

Source		Type III Sum of Squares	<i>df</i>	Mean Square	<i>F</i>	Sig.	Partial Eta Squared
Recipe	Sphericity assumed	4,726.46	2	2,363.23	28.7	0.000	0.76
	Greenhouse–Geisser	4,726.46	1.68	2,807.57	28.7	0.000	0.76
	Huynh–Feldt	4,726.46	2.0	2,363.23	28.7	0.000	0.76
	Lower-bound	4,726.46	1.0	4,726.46	28.7	0.000	0.76
Error(Recipe)	Sphericity assumed	1,480.86	18	82.27			
	Greenhouse–Geisser	1,480.86	15.1	97.73			
	Huynh–Feldt	1,480.86	18.0	82.27			
	Lower-bound	1,480.86	9.0	164.54			

## Hands-on Practice

- Q1.** A management institute wants to determine the best teaching method in order to improve the learning outcome for students. The institute selects one particular topic from course content for undergraduate students and delivers this topic from three different teaching methods (Power Point Presentation (PPT)-1, Simulation-2 and Case study-3) for same set of 20 students on repeat manner. A conceptual-based test has been employed after each method of teaching and marks are recorded out of 30. The data is presented as follows.

<i>Student</i>	<i>PPT</i>	<i>Simulation</i>	<i>Case Study</i>	<i>Student</i>	<i>PPT</i>	<i>Simulation</i>	<i>Case Study</i>
1	12	14	26	11	19	10	21
2	16	19	21	12	23	21	16
3	15	18	15	13	17	14	19
4	23	12	22	14	21	24	27
5	12	15	16	15	22	27	26
6	18	19	21	16	21	18	19
7	21	22	18	17	18	19	20
8	13	16	19	18	28	27	26
9	26	24	29	19	21	12	23
10	15	18	19	20	17	19	21

**Answer the following based on the above-mentioned information:**

- Prepare data set in SPSS Data Editor by using values as mentioned in the case.
- Apply one-way ANOVA repeated measure and comment on Mauchly's Test of Sphericity.
- Comment on the statistical difference among the score of marks.
- Consider different teaching methods as treatment and design of the same experiment in the form of one-way ANOVA. Compute the results and compare with repeated ANOVA.

- Q2.** One advertisement agency wants to examine the effectiveness of three different mediums of an advertisement. The theme of advertisement is to save the water. For this purpose, three advertisements include mobile message, TV commercial and newspaper ads are shown to 10 audiences in a repeated manner under controlled environment. The effectiveness of a particular medium is measured on 10-point Likert scale (1 = least effective and 10 = highly effective). The rating given by the audience for each medium is presented as follows:

	<i>Rating (1 = least preferred and 10 = highly preferred)</i>									
<i>Audience</i>	1	2	3	4	5	6	7	8	9	10
Mobile message	4	6	7	5	6	8	9	10	7	4
TV commercial	3	5	3	8	9	10	7	3	8	9
Newspaper ad	2	1	3	8	9	10	10	7	8	8

**Answer the following by accommodating one-way ANOVA repeated method:**

- Prepare data file in SPSS by identifying suitable variables and using appropriate scales as per the requirement of one-way ANOVA repeated manner.
- Frame suitable hypotheses and examine the difference in effectiveness among three different modes of advertisements.

- c. Compute pairwise differences in the effectiveness among all the mediums.
- d. Comment on the explained and unexplained variance in this repeated ANOVA experiment.

**Q3.** Ten salespersons were appointed one by one in three different residential colonies during their training period. The salespersons were assigned a task to generate orders for 'Child Encyclopedia' book from the selected colonies. The following data pertains to the number of orders received for *Encyclopedia* from these residential areas within one month.

Salespersons	1	2	3	4	5	6	7	8	9	10
<i>Colonies</i>										
Defence	23	12	27	20	12	43	24	32	11	19
Pawanpuri	10	19	21	28	23	13	26	22	24	12
Khaturia	21	30	23	34	26	23	12	34	11	18

**Answer the following questions:**

- a. Prepare data file in SPSS in order to perform one-way ANOVA repeated manner to find out whether the order received from salesperson are same.
- b. Frame suitable hypotheses and interpret the results.
- c. Comment on the explained and unexplained variance in this repeated ANOVA experiment.

## REFERENCES

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